Title: Inscribed Angles, Central Angles, and Their Arcs

Brief Overview:

Many students have difficulty in visualizing the relationships between inscribed arcs, central angles, and the resulting arcs. In this activity, the students will use the TI-92 to discover that the measure of an inscribed angle in a circle is equal to one-half the measure of its intercepted arc, angles inscribed in the same arc are congruent, and that every angle inscribed in a semi-circle is a right angle.

Link to Standards:

- **Problem Solving** Students will be able to solve problems by applying the concepts learned in this lessons.
- **Communication** Students will be able to follow instructions to construct geometric figures with a TI-92 calculator. Students will write and discuss their observations.
- **Reasoning** Students will observe data involving central and inscribed angles, recognize patterns between the two, and make generalizations from their observations, which is the process of inductive reasoning.
- **Geometry** Students will be able to determine the relationships between inscribed angles and their intercepted arcs.

Grade/Level:

Grades 9-12

Duration/Length:

This activity will take 3 days of regular classes or 2 block periods. The activities may take longer than anticipated depending on class duration and student's prior knowledge and familiarity with the TI-92.

Prerequisite Knowledge:

Students should have working knowledge of the following skills or topics:

- Basic use of the TI-92
- Recording data on printed tables
- Central and Inscribed Angles
- Arc and Angle measures

Objectives:

Students will:

- work cooperatively in groups.
- collect and organize data from the TI-92.
- discover the properties of inscribed angles.
- apply the properties of inscribed angles to solve problems.

Materials/Resources/Printed Materials:

- TI-92 calculator
- TI-92 overhead screen
- Overhead projector
- Student Worksheets
- Protractors

Development/Procedures:

- Assign calculators and distribute worksheets.
- Lead students through instructions for constructing a circle, central angle, and an inscribed angle.
- Vary the endpoint of the arc and record data.
- Vary the vertex of the inscribed angle and record data.
- Have students postulate and write a summary of their observations.
- Lead students through instructions for constructing a right angle within a semi-circle.
- Vary the vertex of the right angle and record data.
- Have students postulate and write a summary of their observations.

Evaluation:

Students will complete worksheets at the end of each lesson. Day 1 assignment consists of drawing and measuring central and inscribed angles and summarizing their observations. Day 2 assignment consists of students solving problems by applying their newly learned properties.

Extension/Follow Up:

Activity 1:

Have students measure all three angles of a triangle that is inscribed in a semi-circle. Drag angles and record changes. Discover properties.

Activity 2:

Have students move vertex of the central angle to the side of the circle. Observe the shapes created and discover properties relating to polygons inscribed in a circle.

Activity 3:

Have students create a central angle. Drag the vertex over to the edge of the circle. Record the angle measures during the transition and discover relationship between the angle and the position within and on the circle.

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Circles, Arcs, and Angles

Teacher's Guide

Investigation of arcs, central angles, inscribed angles

Technology

It is ideal to use this learning unit with each student in the class having access to a TI-92 calculator. A maximum of 3 students per calculator is advised. Class sets of 16 calculators can be obtained through a loan program from Texas Instruments. (214) 917-1550. It is also possible to use this unit with only one calculator which is linked to an overhead projector. The teacher will demonstrate as the students observe, comment, discuss, and record.

Keying In

Keystrokes are detailed for the students step by step. Some initial instruction in operating the cursors and function selection may be necessary before beginning.
Before beginning exercise have students clear data tables.

1. Press **APPS,** Select **6** Data/Matrix Editor
2. Press right cursor, select **1** Current. **ENTER.**
3. Press **F1**, select **8** Clear Editor. **ENTER.**

Helpful Hints

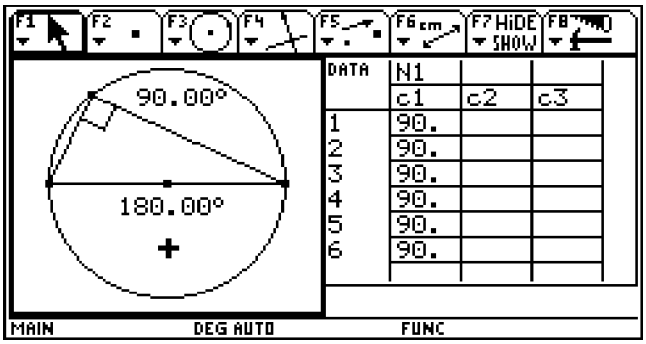
- Make sure the students understand the pointer must say 'this point' or 'on this circle'.
- If the diameter is not measured at exactly 180 degrees it must be redrawn
- If the angle inscribed in the semi-circle is not exactly 90 degrees it must be redrawn.
- Students should verify a right angle has been formed by marking the angle.
- When marking data to be collected a dotted box should appear around each measure.
- When checking the data tables in the TI-92 students may notice a row that has '**und..**', written in the cells. Ask for discussion as to why they think this may have happened. This occurs when the angles whose vertices are the endpoints of diameter measure 90 and 0 degrees, therefore no longer forming a triangle.

Answers to Day 2 Worksheet:

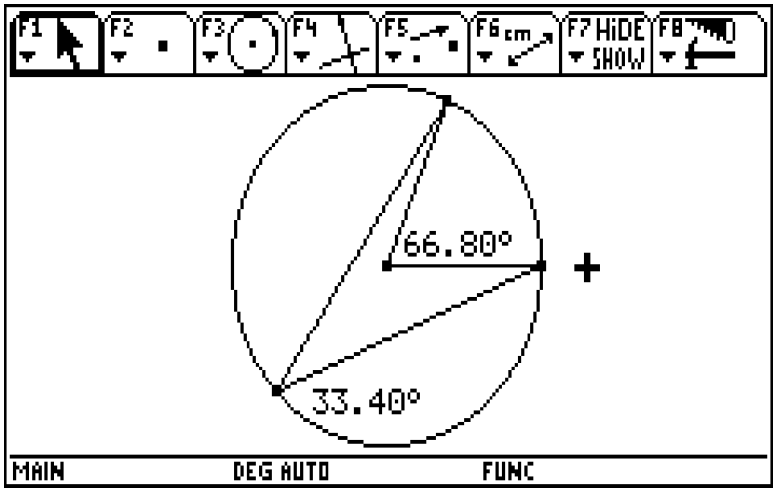
1. X =120
2. C=138
3. Y=72.5
4. X=33
5. Y=140 Z=111
6. X=70
7. Z=128
8. C=45 X=90
9. W=94 X=85 Y=47 Z=96

Included are the graphs captured from the screen of the TI-92 as they should appear at the completion of each set of instructions.

Set 1:



Set 2:



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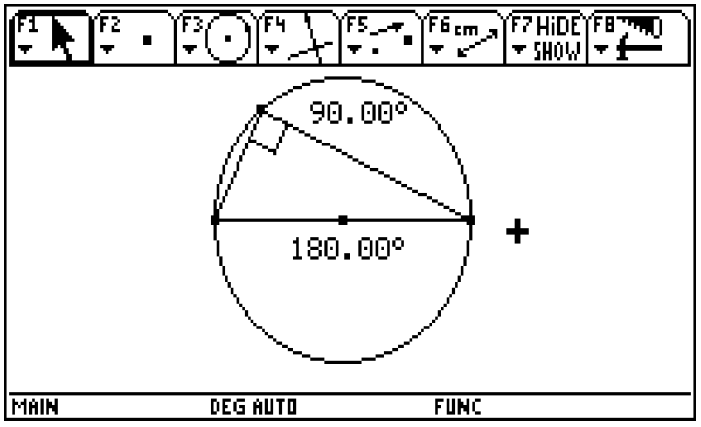


Student Worksheet Circles, Arcs, and Angles name_____

Day 1

Investigation of arcs, central angles, inscribed angles

In class: *Students- sign out a TI92 calculator before you begin this worksheet.*

Recall that the measure of a central angle is equal to its intercepted arc. The reverse of this statement is also true, the measure of an arc is equal to its central angles.
Use the keystrokes described below to do the following:

- *Construct a circle*
- *Place a central angle within the circle*
- *Draw an inscribed angle which intercepts the same arc as the central angle*
- *Determine the measure of the central angle and the inscribed angle*
- *Use the drag function to change the measures of the angles*
- *Record your answers on the sheet provided*
- *Summarize your results and draw a conclusion*

To construct a circle:

1. Press **ON**
2. Press **APPS,** select **8** Geometry
3. Press **F3,** select **1** Circle
4. Move the cursor near center of screen. **ENTER.**
5. Move cursor right to form circle. **ENTER.**

To construct a central angle:

1. Press **F2**, select **5** Segment
2. Place cursor **"on this circle"**. **ENTER.**
3. Move cursor to center of circle "**this point**". **ENTER.**
4. Repeat steps 2 and 3 to form angle.

To construct an inscribed angle that intercepts the arc of central angle just drawn:

1. Press **F2**, select **5** Segment
2. Place cursor to one endpoint of the arc "**this point of intersection**" or "**this point**". **ENTER.**
3. Move cursor to opposite side of circle "**this circle**". **ENTER.**
4. Place cursor at other end of arc "**this point of intersection**" or "**this point". ENTER.**
5. Move curser to point made in step 3 "**this point**". **ENTER.**

To measure the angles:

1. Press **F6,** select **3** Angle
2. Move cursor to endpoint of angle to be measured "**this point**". **ENTER.**
3. Move cursor to vertex of angle to be measured "**this point**".**ENTER.**
4. Move cursor to remaining endpoint of angle to be measured "**this point**". **ENTER**.
5. Repeat the above procedure for the remaining angle.

After completing the previous page of instructions you should have a circle, a central angle, and an inscribed angle on your screen. Both angles should be measured and their measures should be displayed on the screen. If this is not the case **STOP NOW!!!** and ask your teacher for help or press **F8**, select **8** Clear All and begin again.

Once you have successfully completed the above, use the keystrokes described below to vary the size of the central and inscribed angles. Follow these directions **5 times**. Closely observe and record what happens on the worksheet given to you by your teacher during the 5 changes.

Set 1

1. Place cursor on right endpoint of the arc "**this point**".
2. Press and **HOLD DOWN** the 'hand' key.
3. Move cursor right a short interval, release 'hand' key.
4. Record results on worksheet provided.

Repeat for 4 additional intervals.

Summarize your observations in the space provided below the chart on the worksheet and continue.

You will now use the keystrokes described below to vary the position of the vertex of the inscribed angle on the circle. Follow these directions **5 times**. Closely observe and record what happens during the 5 changes on the chart on the worksheet.

Set 2

1. Place cursor on the vertex of the inscribed angle "**this point**".
2. Press and **HOLD DOWN** the 'hand' key.
3. Move cursor right a short interval, release 'hand' key.
4. Record results on worksheet provided.

Repeat for 4 additional intervals.

Summarize your observations in the space provided on the worksheet and continue.

Student Lab Worksheet

Set 1

central angle measure	arc measure	inscribed angle measure	observed relation-ship of central angle to inscribed angle	observed relation-ship of inscribed angle to arc
1.				
2.				
3.				
4.				
5.				

From your observations above write a rule that will always apply to inscribed angles and their arc measures:

Set 2

central angle measure	arc measure	inscribed angle measure	observed relation-ship of central angle to inscribed angle	observed relation-ship of inscribed angle to arc
1.				
2.				
3.				
4.				
5.				

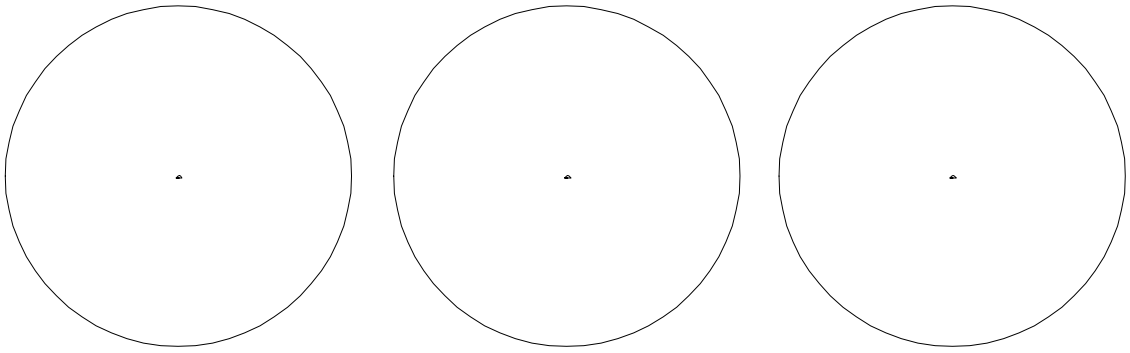
From your observations above write a summary of the effect of changing the vertex on the measure of the inscribed angle:

Homework Worksheet Circles, Arcs, and Angles Name_____

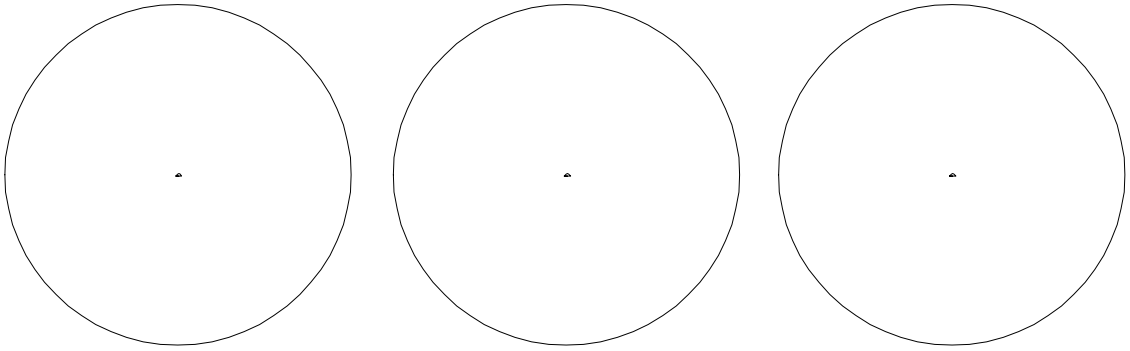
Day 1

Investigation of arcs, inscribed angles, right angles

On each of the circles below draw in a central angle. Use your protractor to measure the angle drawn and the arc it forms. Write on the arc and in the angle the measures found.



On each of the circles below draw in an inscribed angle. Use your protractor to measure the angle drawn and the arc it forms. Write on the arc and in the angle the measures found.



From the first set of circles what did you observe to be true about central angles and arcs?

From the second set of circles what did you observe to be true about inscribed angles and arcs?

Student Worksheet Circles, Arcs, and Angles name_____

Day 2

Investigation of arcs, inscribed angles, right angles

Students- sign out a TI92 calculator before you begin this worksheet.

Use the keystrokes described below to do the following:

- *Construct a circle*
- *Place a diameter within the circle*
- *Draw an inscribed angle which intercepts the endpoints of the diameter*
- *Determine the measure of the inscribed angle*
- *Use the animation function to change the angle's vertex*
- *Observe what occurs carefully*
- *Summarize your results and draw a conclusion*

To construct a circle:

1. Press **ON**
2. Press **APPS,** select **8** Geometry
3. Press **F3**, select **1** Circle
4. Move cursor near center of screen. **ENTER.**
5. Move cursor to right to form a circle. **ENTER.**

To construct a diameter:

1. Press **F2,** select **5** Segment.
2. Move cursor to edge of circle "**on this circle**". **ENTER.**
3. Move cursor through center of circle to other side of circle "**on this circle**". **ENTER.**
   (*Be sure segment passes through center point!)*

Measure straight angle (diameter):

1. Press **F6,** select **3** Angle
2. Move cursor to endpoint of diameter "**this point**". **ENTER.**
3. Move cursor to center of circle "**this point". ENTER.**
4. Move cursor to remaining endpoint of diameter "**this point**". **ENTER.**
   (180 degrees should appear, if it does not then clear and repeat)

Draw inscribed angle in the semi-circle:

1. Press **F2**, select **5** Segment.
2. Move cursor to endpoint of diameter "**this point**". **ENTER.**
3. Move cursor to the edge of the circle "**on this circle**". **ENTER**.
4. Move cursor to other endpoint of diameter "**this point**".**ENTER.**
5. Move cursor to endpoint of first segment "**this point**" to form an angle. **ENTER**.

Measure inscribed angle:

1. Press **F6,** select **3** Angle
2. Move cursor to intersection of angle and diameter "**this point**". **ENTER.**
3. Move cursor to vertex of angle "**this point**". **ENTER.**
4. Move cursor to remaining intersection of angle and diameter "**this point**". **ENTER.**
   (What angle measure appears? _____).

DoubleCheck:

1. Press **F7,** select **7** Mark angle
2. Move cursor to intersection of angle and diameter "**this point**". **ENTER.**
3. Move cursor to vertex of angle "**this point**". **ENTER.**
4. Move cursor to remaining intersection of angle and diameter "**this point**". **ENTER.**
   (What mark appears? Draw it here _____).

After completing the previous page of instructions you should have a circle, a diameter, and an inscribed angle with endpoints on the diameter. The diameter should be measured in degrees and equal 180. The mark from doublecheck should be a right angle mark. If this is not the case **STOP NOW!!!** and ask your teacher for help, or press **F8,** select **8** Clear All and begin again.

Once you have successfully completed the above use the keystrokes described below to vary the position of the vertex of the inscribed angle. Follow these directions **5 times**. Closely observe and record what happens during the 5 changes.

1. Place the cursor on right endpoint of the diameter "**this point**"
2. Press and **HOLD DOWN** the 'hand' key.
3. Move cursor right a short interval, release 'hand' key.
4. Record results below.

Repeat for 4 additional intervals.

Summarize your observations in the space provided and continue.

Angle in a semi-circle

straight angle (diameter) measure	inscribed angle measure	change in inscribed angle	observed relationship of straight angle to inscribed angle
1.			
2.			
3.			
4.			
5.			

From your observations above write a summary of the effect of changing the vertex on the measure of an angle inscribed in a semi-circle:

Make a general observation about the measure of angles inscribed in semi-circles.

Animation Activity

Use the keystrokes described below to do the following:

- *Create a table*
- *Move the vertex continuously around the circle*
- *Store the results in a table*
- *Observe and summarize the results in the table*

Store data in a table:

1. Press **F6**, select **7** Collect Data
2. Press right cursor key, select **2** Define Entry. **ENTER.**
3. Move cursor to vertex angle. **ENTER.**
4. Press **F6,** select **7** Collect Data
5. Press right cursor key, select **1** Store Data. **ENTER.**

Animation:

1. Move cursor to vertex of inscribed angle.
2. Press **F7**, select **3** Animation. **ENTER.**
3. Press and **HOLD DOWN** the 'hand' key.
4. Move cursor right a short interval, release 'hand' key.
5. After one complete revolution press **ON** key until motion stops.

Summarize below what you observed:

Check Data Table:

1. Press **F8**, select **B** Data View. **ENTER.**

Summarize the table contents:

Clear Data Table:

1. Press **F8**, select **C** Clear Data View. **ENTER.**

What seems to always be true about angles drawn in a semi-circle?

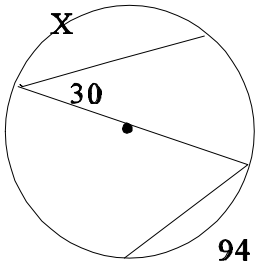
Homework Worksheet Circles, Arcs, and Angles Name_____

Day 2

Investigation of arcs, inscribed angles, right angles

Use the properties of central angles, inscribed angles, triangles, and diameters to solve for x in each of the following:

1.



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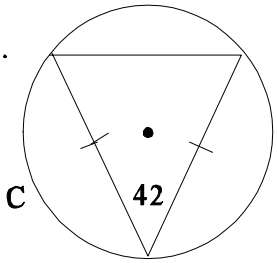

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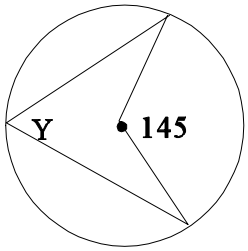

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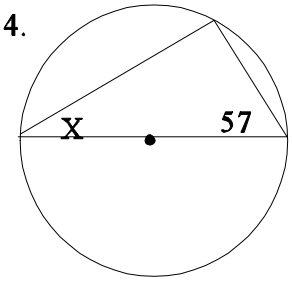

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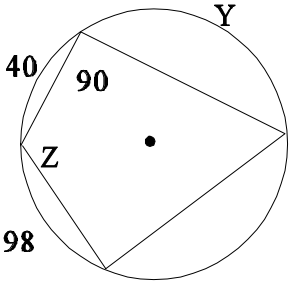

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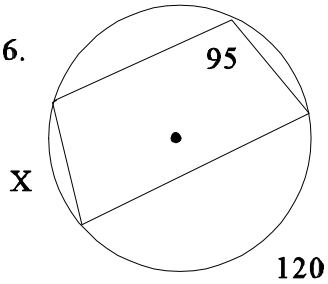

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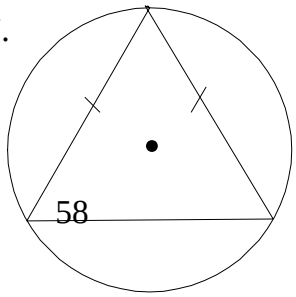

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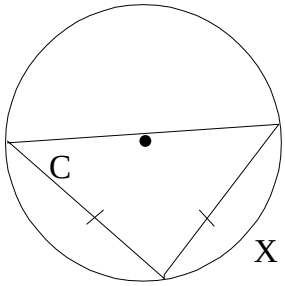

9.